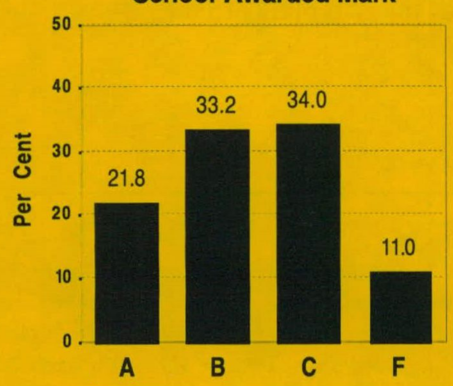
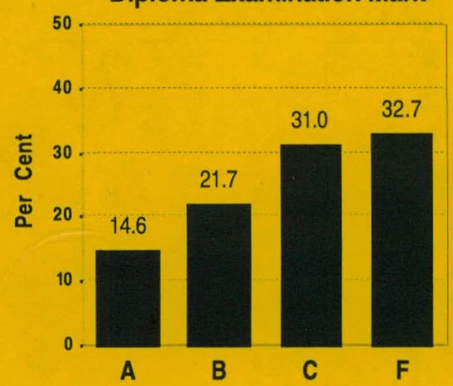
Mathematics 30

Diploma Examination Results
Examiners' Report for June 1993

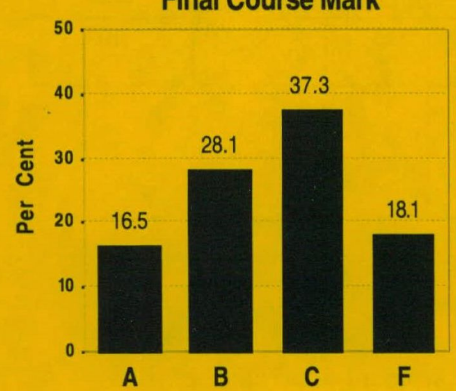
School-Awarded Mark



Diploma Examination Mark



Final Course Mark



The summary information in this report provides teachers, school administrators, students, and the general public with an overview of results from the June 1993 administration of the Mathematics 30 Diploma Examination. This information is most helpful when used with the detailed school and jurisdiction reports that have been mailed to schools and school jurisdiction offices. An annual provincial report containing a detailed analysis of the combined January, June, and August results is published each year.

Description of the Examination

The Mathematics 30 Diploma Examination consists of three parts: a multiple-choice section of 42 questions worth 60%, a numerical-response section of seven questions worth 10%, and a written-response section of four questions worth 30% of the total examination mark.

Achievement of Standards

The information reported is based on the final course marks achieved by 10 676 students who wrote the June 1993 examination.

- 81.9% of these students achieved the acceptable standard (a final course mark of 50% or higher).
- 16.5% of these students achieved the standard of excellence (a final course mark of 80% or higher).

For the fifth year in a row, approximately 30% of the students failed to meet the acceptable standard on June examinations. Students can perform mathematical procedures but have difficulty in demonstrating their understanding of mathematical concepts and in problem-solving.

Provincial Averages

- The average school-awarded mark was 66.4%.
- The average diploma examination mark was 58.3%.
- The average final course mark, representing an equal weighting of the school-awarded mark and the diploma examination mark, was 62.7%.

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During the production process, a typographical error occurred in numbering multiple-choice questions 26 and 27. The questions were in the right order, but the question numbers were reversed. Some students corrected the misprint and recorded their responses on the answer sheet accordingly; others matched the question numbers as printed with those numbers on the answer sheet. To ensure students' responses were marked accurately and fairly, we scored the student's answer sheet twice, first using a marking key that assumed one of the two ways of recording responses and then using a marking key that assumed the other way. Then, we allocated the higher of the two marks to the student.

Results and Examiners' Comments

Subtest

When analyzing detailed examination results, please bear in mind that subtest results **cannot** be directly compared.

Results are in average raw scores.

Machine scored: 30.0 out of 49

Written response: 10.7 out of 21

Course Content

- Polynomial Functions: 7 out of 9
- Trigonometric and Circular Functions: 7.9 out of 13
- Statistics: 7.5 out of 13
- Quadratic Relations: 3.8 out of 9
- Exponential and Logarithmic Functions: 5.9 out of 9
- Permutations and Combinations: 4.3 out of 8
- Sequences and Series: 5.5 out of 9

Cognitive Level *

- Procedural: 12.1 out of 17
- Conceptual: 8.5 out of 15
- Problem Solving: 9.4 out of 17

*Refer to Appendix D of the 1992–93 *Mathematics 30 Diploma Examinations Program Bulletin* for an explanation of cognitive levels.

Note: The written-response section spans all cognitive levels.

Examination Blueprint

Each question on the examination is classified in two ways: according to the curricular content area being tested and according to the cognitive level. The examination blueprint illustrates the distribution of questions in June 1993 according to these classifications. In the machine-scored columns, numbers without parentheses are multiple-choice queions and those in parentheses ( ) are numerical-response questions.

Course Content	**Machine Scored**			**Written Response**	**Examination Emphasis (%)**
	Procedural	**Conceptual**	**Problem Solving**		
Polynomial Functions	(1)	13	14,15	4	13
Trigonometric and Circular Functions	1,3,4,5,11	2,6,8,10	7,9,12,(6)		18.5
Statistics	17,18	16,(3)	18,20	3	18.5
Quadratic Relations	(2)	21	22,23	2	13
Exponential and Logarithmic Functions	24,25,29	26,27,30	28,31,(7)		13
Permutations and Combinations	33,34,36	32,37	35,38,(5)		11
Sequences and Series	39,40	41,(4)	42	1	13
Examination Emphasis(%)	24.3	21.4	24.3	30	100

This examination has a balance of question types and difficulties reflecting the philosophy of the Mathematics 30 Course of Studies. It is designed so that students capable of achieving the acceptable standard will obtain a mark of 50% or higher and students capable of achieving the standard of excellence will obtain a mark of 80% or higher.

Future examinations will continue to focus on assessing students' understanding mathematical concepts and on problem solving. Students will continue to be

expected to solve problems, explain solutions, justify solutions, or prove solutions in the written-response section. The design of future field tests and examinations will include items that assess how well students have achieved the general learner expectations stated in the Mathematics 30 Course of Studies.

Multiple Choice

Question	Key	Difficulty**	Question	Key	Difficulty**
1	B	83.5	22	C	56.6
2	A	69.6	23	D	36.2
3	B	79.8	24	B	88.9
4	B	67.9	25	C	84.8
5	D	65.5	26†	B	91.3
6	C	80.6	27†	D	41.4
7	D	40.3	28	C	60.0
8	A	42.8	29	B	70.2
9	C	64.8	30	A	41.9
10	C	35.9	31	A	75.9
11	A	53.1	32	A	28.1
12	C	67.2	33	D	79.7
13	B	89.7	34	D	47.2
14	B	67.6	35	B	41.8
15	A	49.1	36	C	63.9
16	C	49.0	37	D	49.4
17	A	44.6	38	D	46.3
18	C	63.5	39	B	74.4
19	A	85.2	40	C	69.4
20	A	64.4	41	B	63.9
21	D	48.2	42	D	56.4

** Difficulty—percentage of students answering the question correctly
†These questions were numbered on the exam as 27 and 26.

Questions 9, 11, 12, 15, 18, and 35 were classified as questions that students who achieve the standard of excellence could successfully answer. Students achieving the acceptable standard were expected to correctly answer the remaining questions. Two illustrative questions follow:

5. Correct to the nearest degree, the solution to $3\cos\theta + 1 = 0$, $0^\circ \le \theta < 360^\circ$, is

   A. 79°
   B. 109°
   C. 79° and 289°
   •D. 109° and 251°

Question 5 required students to determine the solution to a first-degree trigonometric equation, recognizing that two solutions exist in the stated domain. It is expected that all students who achieve the acceptable standard in Mathemathics 30 can solve first-degree trigonometric equations on the domain $0^\circ \le \theta < 360^\circ$. Approximately 72% of those students who met the acceptable standard on the examination were able to do this and chose alternative D. If students could solve the equation but did not realize that there were two solutions on the stated domain, they chose alternative B, and 14% of all students who met the acceptable standard on the examination chose this alternative. Those students who recognized that there were two solutions to the equation but made an error in their solution chose alternative C. Approximately 8% of the students who met the acceptable standard chose alternative C. Of the students who met the standard of excellence on the examination, 91% were able to correctly identify alternative D.

37. A child is playing with 4 blocks: one red, one blue, one orange, and one green. In how many different ways can the child stack 2, 3, or 4 of these blocks?

A. 12

B. 16

C. 24

•D. 60

Question 37 required students to calculate the number of different ways that 2, 3, or 4 blocks can be stacked when given four blocks, each of a different color. Students could either use the Fundamental Counting Principle or linear permutations to perform this calculation. Students performing at an acceptable standard in Mathematics 30 are expected to be able to calculate the number of linear, circle, and ring permutations of n things taken r at a time. Approximately half of the students, 53.2%, who met the acceptable standard but not the standard of excellence on the examination were not able to meet the acceptable standard on this question. These students could correctly calculate the number of ways of arranging only four blocks, with no consideration for arranging two or three blocks, and chose alternative C. Of the students who met the standard of excellence, on the examination, 81.2% answered correctly.

Numerical Response

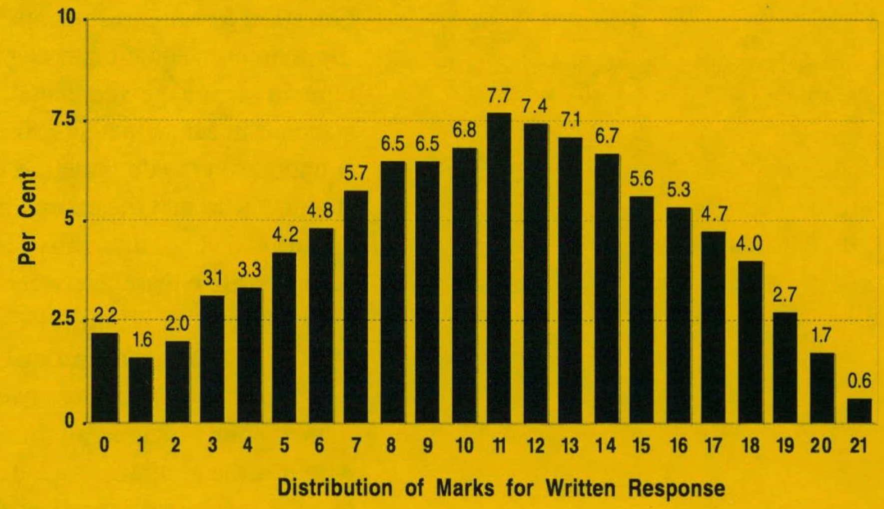
Question	Key	Difficulty**
1	5	75.4
2	90	74.6
3	63.8	72.1
4	4465	44.1
5	10	76.0
6	68.8	38.8
7	8	39.6

**Difficulty—percentage of students answering the question correctly

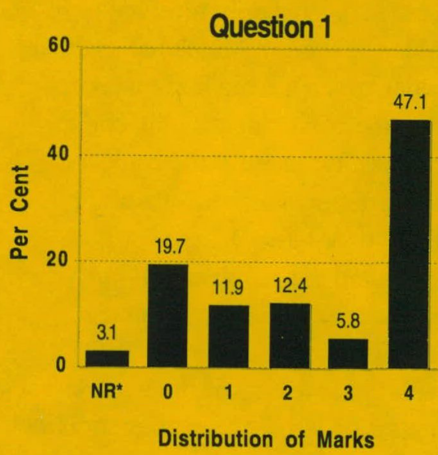
Questions 1, 6, and 7 were classified as questions that students who achieve the standard of excellence could successfully answer.

Written Response

A distribution of total raw scores received on the written-response section of the exam follows:



Questions in the written-response section dealt with four of the seven content strands for Mathematics 30. Students performing at the acceptable standard were expected to obtain at least half marks on all questions. Students performing at the standard of excellence were expected to answer all questions almost perfectly.



*NR—No Response

Question 1 required students to determine the number of days that had passed before a parking ticket was paid. This question required students to recognize that this pattern formed a geometric sequence. Some students applied the compound interest formula, $A = P(1 + i)^n$ and determined 'n'; some students determined the nth term in a geometric sequence; still other students wrote out the terms in the sequence. If students carried through their strategy and arrived at an answer of 232 or 233 days, they received 4 marks. Students who meet the acceptable standard in Mathematics 30 are expected to describe, orally and in writing, the difference between sequences and series, arithmetic or geometric, infinite and finite; apply the general term formula for arithmetic and geometric sequences; and participate in and contribute toward the problem-solving process for problems involving sum and term formulas for arithmetic and geometric series and sequences. Based on these standards, it was expected that students who met the acceptable standard would achieve 3 or 4 marks on this question. Of all students who met the acceptable standard on the examination, only 64.9% achieved 3 or 4 marks on this question.

On this 4-mark question, the average mark was 2.42 or 60.5%.

Scoring Question 1: The scoring guide for question 1 was adapted from the Focused Holistic Scoring Point Scale, p. 35, in *How to Evaluate Progress in Problem Solving*, published by the National Council of Teachers of Mathematics, 1987.

Scoring Guide for Written-Response Question 1

4 marks: The student decides on an appropriate strategy and carries through that strategy with a correct and complete procedure. The student determines it took 232 or 233 days for Joseph to pay his ticket, depending on the strategy chosen.

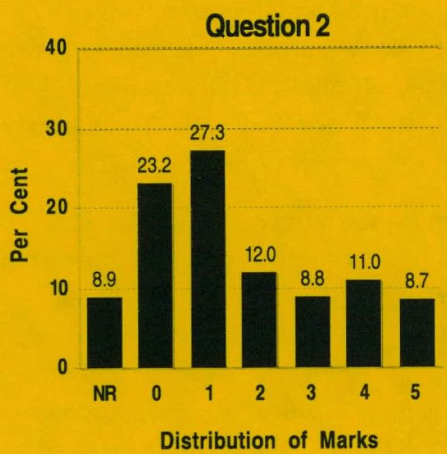
3 marks: The student decides on an appropriate strategy and carries through the complete procedure, obtaining the incorrect answer.

2 marks: The student demonstrates some understanding of the problem, selects and begins a strategy related to geometric sequences; for example, correctly determines 2 of the 3 unknowns ($A, P, i; a, r, t_n$) in a formula and begins to determine a solution. Note: If students use a geometric sum, then $S_n = \$90.60$, $a = 0.1$, $r = 1.01$).

1 mark: The student demonstrates some understanding of the problem OR shows in some way that the sequence formed is geometric. For example, only a formula may be identified OR enough terms are written to clearly indicate that the sequence is geometric but proceeds to solve it as an arithmetic sequence OR the answer, 232 or 233 days, is given with little or no justification, e.g., "I used my calculator to arrive at 232 days."

0 marks: The student solves the problem with the sequence as arithmetic.

Note: The acceptable standard states students are able to recognize geometric and arithmetic sequences; hence, no mark will be given for claims that the sequence is arithmetic.



Question 2 required students to determine a quadratic relation, given some information, and then decide which equation describes the quadratic relation. To receive full marks, students needed to clearly explain how they reached their conclusion. Many students believed that the only quadratic relation with a directrix is the parabola and hence identified Equation A as the correct equation . These students only received only 2 marks. Students who meet the acceptable standard in Mathematics 30 are expected to calculate the eccentricity when given a fixed horizontal or vertical line, a fixed point, and a point on the conic; identify the conic formed when given the value of the eccentricity; identify the conic defined by a combination of numerical coefficients for any quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, $B = 0$; and participate in and contribute toward the problem-solving process for problems that require the analysis of quadratic relations studied in Mathematics 30. Hence, students who are achieving the acceptable standard were expected to achieve at least 3 of the 5 marks for this question. Only 28.5% of the students who wrote the examination met or exceeded this. It is expected that students who meet the standard of excellence can complete the solution to problems that require the analysis of quadratic relations; 38.7% of the students who met the standard of excellence on the examination achieved full marks on this question.

On this 5-mark question, the average mark was 1.65 or 33.0%.

Scoring Question 2: The scoring guide for question 2 was adapted from the Focused Holistic Scoring Point Scale, p. 35, in *How to Evaluate Progress in Problem Solving*, published by the National Council of Teachers of Mathematics, 1987.

Scoring Guide for Written-Response Question 2

5: The student indicates that Equation B is the equation that defines the quadratic relation and provides evidence through a clear explanation.

4: The student's explanation is missing one piece of pertinent information:

e.g., the student explains why the quadratic relation is an ellipse but then states that Equation A is a parabola and/or states that Equation B is an ellipse without supporting evidence.

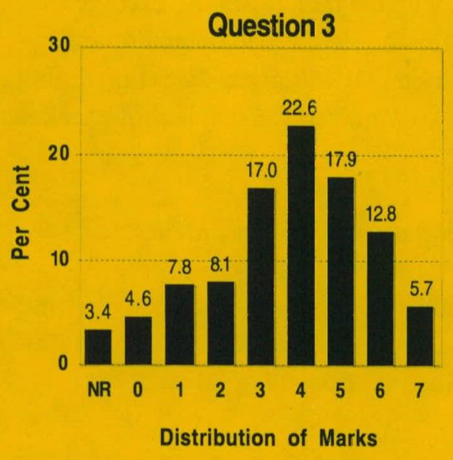
e.g., the student correctly shows that Equation A is a parabola and/or Equation B is an ellipse and identifies the quadratic relation as an ellipse but does not make the connection between the equation and all the given information.

3: The student writes a correct conclusion based on correct but insufficient evidence. E.g., calculates $e = \frac{4}{5}$ and states that equation B is the equation of the quadratic relation. In this example, the student does not identify why the quadratic relation is an ellipse or why equation B is an ellipse. OR the student makes an error in determining the shape of the graph defined by Equation B and provides a complete argument leading to an incorrect conclusion.

2: The student determines that Equation A or Equation B is the correct equation and provides a plausible rationale, although incorrect.

1: The student shows a piece of correct information about Equation A, EquationB, or the quadratic relation without repeating the information given in the question. OR the student shows a sketch with all information about the quadratic relation correctly identified.

0: The student identifies "Equation A" or "Equation B" as the equation that describes the quadratic relation.



Question 3, parts a and b, required students to determine the 90% confidence intervals for two sample sizes, given the result of a survey of Alberta high school students, and then identify a factor that might contribute to differences between the results of the same survey in one high school in the province and the results of the provincial survey. Students who meet the acceptable standard in Mathematics 30 are expected to use charts of 90% box plots to find the confidence interval within which survey results can be interpreted; and assess the strengths, weaknesses, and biases of samples. Students who meet the acceptable standard were expected to score 4 or 5 marks out of 7 on this question, 3 marks on part a and 1 or 2 marks on part b, and students who meet the standard of excellence were expected to score 6 or 7 marks. Of the students who met the acceptable standard, but not the standard of excellence, 67.1% scored at least 4 marks on this question. Approximately 43% of the students who met the standard of excellence scored 6 or 7 marks on this question.

On this 7-mark question, the average mark was 3.72 or 53.1%.

Scoring Question 3: *Part a* was worth 3 marks, which were determined as follows:

Scoring Guide for Written-Response Question 3a

3: The student correctly identifies both confidence intervals using correct units.

2: The student correctly identifies both confidence intervals without units OR correctly identifies 1 confidence interval with the corresponding units OR correctly identifies both confidence intervals using correct units but includes information that demonstrates misunderstanding.

1: The student gives a partial answer, correctly identifying 1 confidence interval with no units.

Part b was worth 4 marks, which were determined as follows:

Scoring Guide for Written-Response Question 3b

4: The student identifies one of the appropriate factors and correctly explains how it affects the results. Two of the appropriate factors are sample size and bias.

E.g., Bias: The bias is that the students at one high school do not represent the entire high school student population in Alberta.

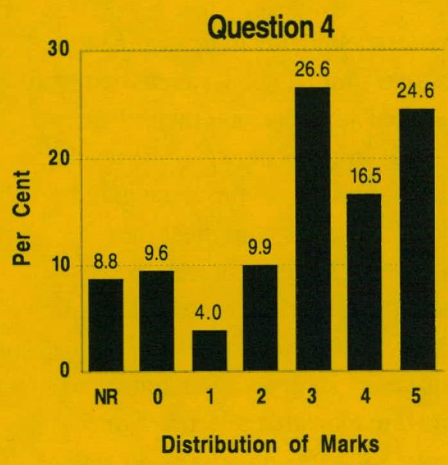
Sample size: students need to compare the sample size from the high school to the sample size of the original survey and discuss how this affects confidence intervals.

3: The student identifies an appropriate factor with an incomplete explanation OR an appropriate factor is implied from a thorough explanation..

2: The student identifies an appropriate factor and includes an invalid assumption OR implies an appropriate factor with an incomplete explanation.

1: The student identifies an appropriate factor with no explanation OR implies an appropriate factor and includes invalid assumptions.

E.g., An implied bias is that the characteristics of the sample are different from the characteristics of the population.



Question 4 required students to determine the equation of the graph of a third-degree polynomial function. All students who meet the acceptable standard in Mathematics 30 are expected to derive an equation of an integral third-degree polynomial function given its rational zeros and find approximations for all the real zeros of integral polynomial functions using graphing calculators or computers. Students who meet this standard should have achieved 3 of the 5 marks. Approximately 80% of the students who met the acceptable standard, but not the standard of excellence, met or surpassed this expectation. Students who are demonstrating excellent achievement can derive an equation for an integral polynomial function given its zeros and any other information that will uniquely define it. In fact, approximately 90% of the students who achieved the standard of excellence met this expectation.

On this 5-mark question, the average mark was 2.92 or 58.4%.

Scoring Question 4: The scoring guide for question 4 was adapted from the Focused Holistic Scoring Point Scale, p. 35, in *How to Evaluate Progress in Problem Solving*, published by the National Council of Teachers of Mathematics, 1987.

Scoring Guide for Written-Response Question 4

5: The student correctly uses one of the processes described to determine

$P(x) = \pm 2(x + 3)(x + 2)(2x \pm 3)$ or other variations

Note: Students were not deducted marks if they did not write an equation as a final answer or if they made an error in multiplying factors, as this does not indicate that they misunderstand the process and strategies.

4: The student uses a correct process that could have led to the correct equation if implemented properly. For example, the student determined incorrect values for the y-intercept or determined the wrong value of the rational zero or found the equation using a point not on the curve OR the student in some way shows recognition that there is a family of curves.

3: The student determines a single solution for the equation, such as

$P(x) = (x + 3)(x + 2)(2x - 3)$ or other variations.

OR The student demonstrates an understanding that more than one polynomial can have the same roots, although incorrectly identifies the relationship between the x-intercept of the graph and the factors of $P(x)$. For example, the student determines that

$P(x) = a(x \pm 3)(x \pm 2)\left(x + \frac{3}{2}\right)$ or other variations.

OR The student recognizes that more than one polynomial can have the same roots, correctly identifies the relationship between the x-intercepts of the graph and the factors of $P(x)$, but incorrectly identifies an x-intercept. For example,

$P(x) = a(x + 3)(x + 2)(x - \frac{1}{2})$ or other variations.

OR The student begins working through solution 2 and establishes 3 equations.

2: The student does not show the relationship between an x-intercept of the graph and the factors of $P(x)$. For example,

$P(x) = (x - 3)(x - 2)\left(x + \frac{3}{2}\right)$

OR The student identifies the relationship between the x-intercepts of the graph and the factors of $P(x)$ but incorrectly identifies an x-intercept. For example,

$P(x) = (x + 3)(x + 2)(2x - 1)$

OR The student begins working through solution 2 and establishes 2 equations.

1: The student demonstrates some understanding of the form of the equation of a third-degree polynomial.

For further information, contact Florence Glanfield or Phill Campbell at the Student Evaluation Branch, 427-0010.